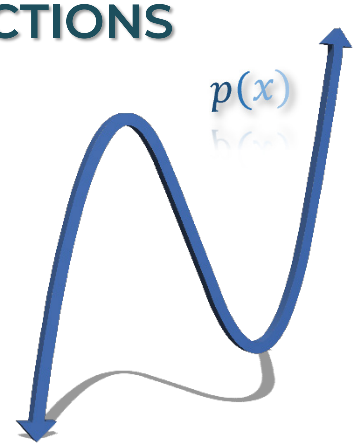
Unit 2

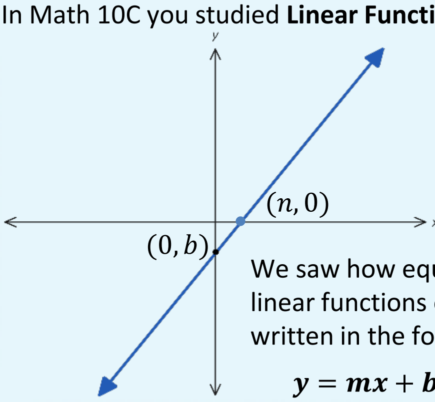
POLYNOMIAL FUNCTIONS



2.1 Characteristics of Polynomial Functions

You've already studied Polynomial functions! *Remember?*

In Math 10C you studied **Linear Functions**



We saw how equations of linear functions can be written in the form

$$y = mx + b$$

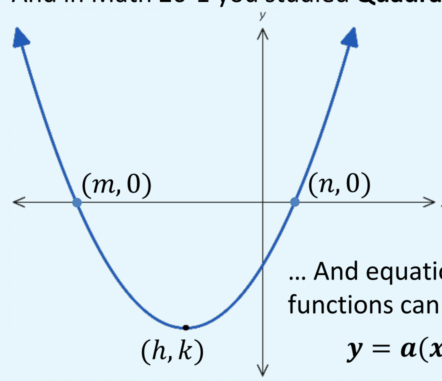
*Where m is the **slope** of the line, and b is the **y-intercept***

Note that the linear functions can also be written in the form $y = m(x - n)$

*Where n is the **x-intercept***

➡ These are **degree 1** Polynomial Functions

And in Math 20-1 you studied **Quadratic Functions**



... And equations of quadratic functions can be written

$$y = a(x - h)^2 + k$$

*Where a is the **vertical stretch**, and the coordinates of the **vertex** are (h, k).*

Note that the quadratic functions can also be written in the form $y = a(x - m)(x - n)$

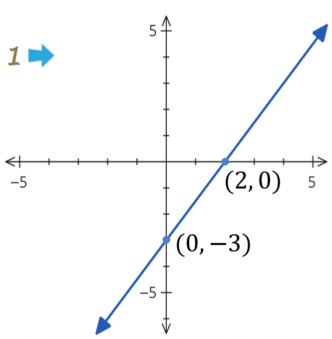
*Where m, n are **x-intercepts***

➡ These are **degree 2** Polynomial Functions

warm up

Determine an equation for each of the following functions:

1



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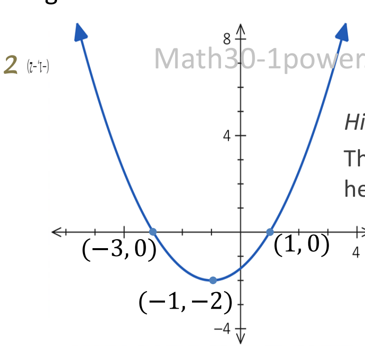


Hint: The vertical stretch here, a, is $1/2$

Defining Polynomial Functions

A **polynomial function**, such as a linear function, quadratic, or cubic, involves only non-negative integer powers of x.

Any polynomial function can be written in the form

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ \ldots + a_2x^2 + a_1x^1 + a_0$$

Where:
- *n is a whole number, representing the degree of the function*
- *a_n to a_0 are real numbers, representing the coefficients, with a_n designated the **leading coefficient** (coeff. of the highest degree term)*

This *general formula* may look complicated, but a few **polynomial function examples** should show its simplicity:

- $y = 8$ — This is **degree 0** (constant function)
- $y = -5x + 4$ — This is **degree 1** (linear), with a leading coefficient of -5
- $f(x) = 2x^2 - 4x + 3$ — This is **degree 2** (quadratic), with a leading coefficient of 2
- $y = x^3 + \frac{1}{2}x^2 - 7x - 1$ — This is **degree 3** (quadratic), with a leading coefficient of 1
- $p(x) = 3x^4 - \sqrt{2}x^3 + 5x - 8$ — This is **degree 4** (quartic), with a leading coefficient of 3
- $p(x) = -2x^5 + x^4 - 3x^2 - 6$ — This is **degree 5** (quintic), with a leading coefficient of -2

These examples are all written in **descending order of degree**, where terms are arranged starting with the highest degree term, starting with the **leading coefficient**. (The coefficient of the highest degree term)

Worked Example

Identify which of the following are polynomial functions. For each that is a polynomial function; state the degree and leading coefficient:

(a) $y = -6x^2 + \frac{3}{x} - 8$ (b) $y = 3x^4 - 2x^5 - 3x^2 - 1$ (c) $y = 5x^2 - 3\sqrt[3]{x} + 1$

Solution:

(a) The middle term can be written $3x^{-1}$, which is **NOT** POLYNOMIAL as exponent of x is not a whole number

(b) All exponents are whole numbers, and all coefficients are real numbers. Hello, you **POLYNOMIAL FUNCTION**. Degree is **5** *(degree of entire poly function is that of highest degree term)*. Leading coefficient is $\mathbf{-2}$. *(the terms are not in descending order of degree – the 2nd term should be re-arranged to the "front"!)*

(c) The middle term can be written $3x^{1/3}$, which is **NOT** POLYNOMIAL as the exponent is not a whole number

Class Example 2.11 *Identifying Polynomial Functions*

Identify which of the following are polynomial functions. For each that is a polynomial function; state the degree and leading coefficient:

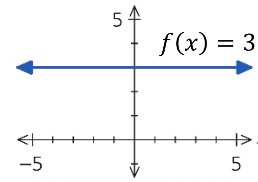
(a) $y = 4x^5 - 3x^2$ (b) $y = 2\sqrt{x} + 5x$ (c) $y = \sqrt{3}\,x + 7$ (d) $y = -2x^3 + 5x^{-1}$

note

Polynomial functions can be of any whole number degree n – but for this course we'll only deal with functions where $n \leq 5$.

And while the coefficients ***can*** be any real number – we'll mainly stick with **integer** coefficients.

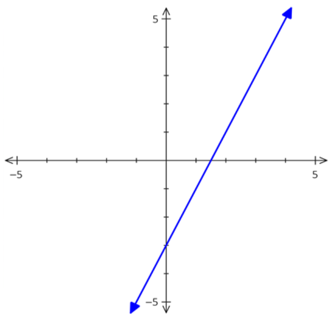
Classifying Polynomial Functions by Degree



A **polynomial function** can be of any whole number degree, including zero!

The graph on the right is of the constant function $f(x) = 3$, which is a polynomial function of degree zero.

Let's now acquaint ourselves with **some examples of polynomial functions**, of degree 1 through 5:



Degree 1 *Linear*

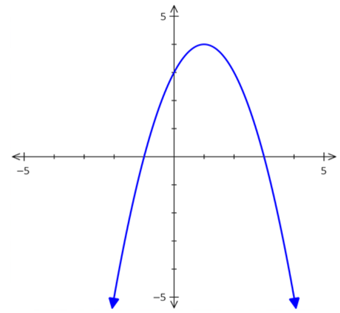
$f(x) = 2x - 3$

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R}\}$

End Behavior:
starts negative in quad III,
ends positive in quad I

# of intercepts: 1



Degree 2 *Quadratic*

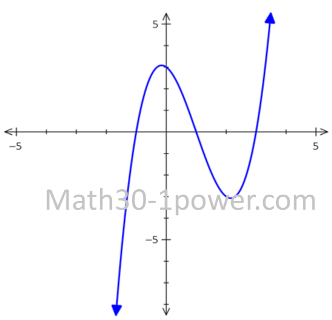
$g(x) = -x^2 + 2x + 3$

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \le 4, y \in \mathbb{R}\}$

End Behavior:
starts negative in quad III,
ends negative in quad IV

# of intercepts: 2



Degree 3 *Cubic*

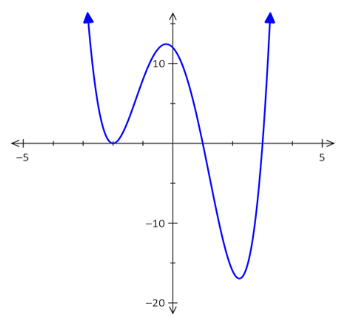
$k(x) = x^3 - 3x^2 - x + 3$

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R}\}$

End Behavior:
starts negative in quad III,
ends positive in quad I

# of intercepts: 3



Degree 4 *Quartic*

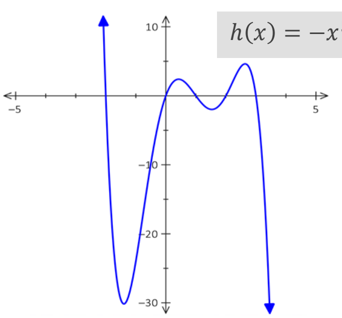
$p(x) = x^4 - 9x^2 - 4x + 12$

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \ge -16.9, y \in \mathbb{R}\}$

End Behavior:
starts positive in quad II,
ends positive in quad I

# of intercepts: 3



Degree 5 *Quintic*

$h(x) = -x^5 + 4x^4 + x^3 - 16x^2 + 12x$

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R}\}$

End Behavior:
starts positive in quad II,
ends negative in quad IV

# of intercepts: 5

➔ The functions on the left are **odd degree** – and the graphs start and end in the opposite direction. *For example, the degree 5 function starts positive and ends negative.*

Odd functions have no max or min point, must have at least one x-intercept, and have a **range** $\{y \in \mathbb{R}\}$.

➔ The functions above are **even degree**. As such, the graphs start and end in the same direction. *For example, the degree 4 function starts positive and ends positive.*

Even functions have either a maximum or minimum point, and the **range** is restricted accordingly.

If the **sign of the leading coefficient is positive** (the degree 1, 3, and 4 examples above), the graph "ends positive", or heading upward, in quad I. And If the degree is even, the graph will have a **minimum point**.

Ends positive

If the sign is **negative** (see the degree 2 and 5 examples), the graph "**end negative**", or downward, in quad IV. And if its even degree (as with example 2), the graph will have a **maximum point**.

I wish my lead coeff. wasn't so negative

Answers from previous page

2.11 (a) For both terms, exponents of x are whole numbers. **POLYNOMIAL FUNCTION** , degree **5** Leading Coeff. **4**

(b) The first term could be written as $2x^{1/2}$, which is **NOT POLYNOMIAL** as the exponent of x is not a whole number.

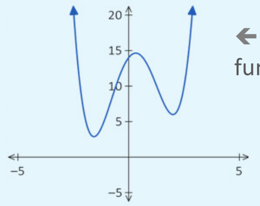
(c) That's a **POLYNOMIAL FUNCTION** (Don't be thrown off by the irrational $\sqrt{3}$ *coefficient* – that's allowed!)
Polynomial functions must have whole # variable exponents, coefficients can be any real #. degree **1** Leading Coeff. $\sqrt{3}$

(d) The last term is **NOT POLYNOMIAL** as the exponent of x is not a whole number.

There is a relationship between the degree of a polynomial function and the number of x-intercepts on the graph.

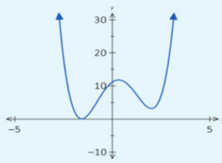
For a polynomial function of degree n; the **maximum** number of x-intercepts is $\boldsymbol{n}$.

- For even degree functions, there can be **0 to $\boldsymbol{n}$** x-intercepts.

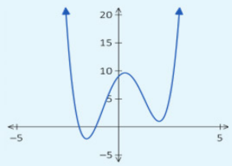


← *For example*, this degree 4 function has *no x-intercepts*

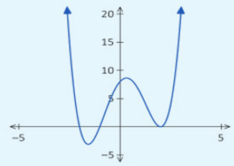
Now, a degree 4 function can also have **1 x-intercept**



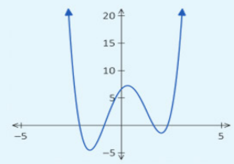
… or **2 x-intercepts**



… or **3 x-intercepts**

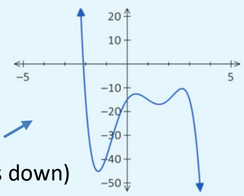


To a maximum of **4**



- For odd degree functions, there can be **1 to $\boldsymbol{n}$** x-intercepts.

Note (unlike the functions above), this function must have a **negative leading coefficient**. (ends down)



← *For example*, consider a degree 5 function

Since it starts / ends in the opposite direction (in this case starts positive in quad II, and negative in quad IV)

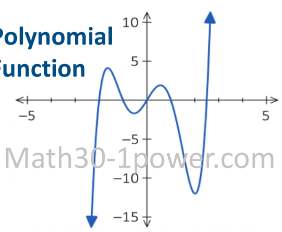
…there must be ***at least one x-intercept***

And we should know how to spot a Polynomial Function Graph!

On the previous page we saw the relationship between the degree of a polynomial function and certain characteristics of the graph. You might next ask – how can we immediately tell that a graph is of a polynomial function, and not some other function we study in Math 30?

And once again – great question! I respect your inquisitive nature.
Let's dive into that, with a couple of key distinguishing points:

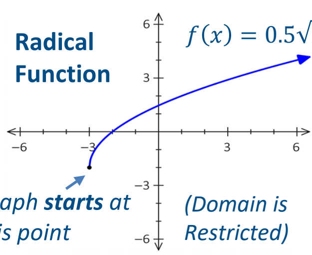
Polynomial Function



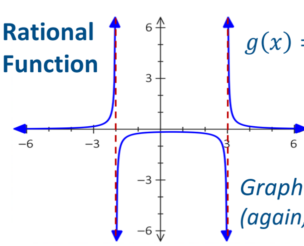
Graph can be drawn without lifting your pencil.

- The first key point is that all polynomial functions have a domain $\{x \in \mathbb{R}\}$. That means graphs of polynomial functions:
  - *Have no* ***start*** *or end* ***points****, like, for example, radical function graphs.*
  - *Have no vertical asymptotes or any other type of discontinuity, as with rational function graphs.*

Radical Function



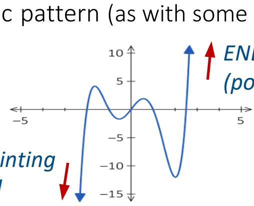
Rational Function



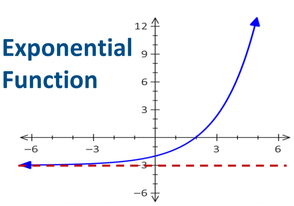
- The second point is polynomial function graphs have no horizontal asymptotes (like exponential functions) and there is no periodic pattern (as with some trig graphs).

  So graphs will always both start and end in either an upward or downward position.

  For example, this graph….



Exponential Function



Worked Example

For the polynomial function $p(x) = -x^4 + 3x^3 + 7x^2 - 15x - 18$;

Without using your graphing calculator, state:

i - The start and end behavior of the graph   ii - The number of possible x-intercepts

iii - Whether or not the graph will have a minimum or maximum point

iv - The domain of the function and the y-intercept

Use your graphing calculator to determine:

v - The x-intercepts of the graph   vi - The range of the function

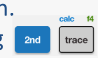
Sol.:

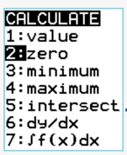
i - The degree of the function is 4; since it's even, the graph will start and end in the same direction. And since the leading coefficient is negative (that is, "-1"), the graph will end negative / heading downward. So…. The graph **starts negative in quadrant III**, and **ends negative in quadrant IV**.

ii - Degree 4 (even), so there can be **between 0 and 4 x-intercepts**.

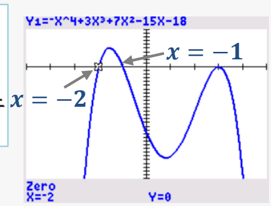
iii - Even degree, graph starts / ends in the same direction. Negative leading coefficient, so which means the graph ends negative. Therefore the graph will **have a maximum point,** which can be found graphically.

iv - All polynomial functions have domain $\{x \in \mathbb{R}\}$. The y-intercept is the same as the constant value, so $(\mathbf{0}, \mathbf{-18})$

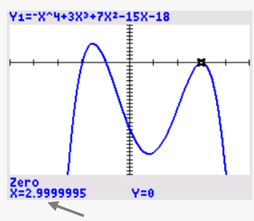
v - x-intercepts are the same as the zeros of the function. The zero function is in CALC menu, found be entering 



RTD Learning
PowerMath



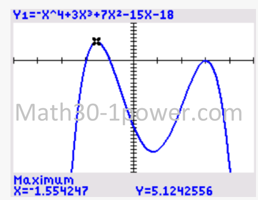
Find the zeros one at a time…



Note: sometimes the calc *adds decimals*. Here, the actual value is just $\mathbf{3}$.

So, **x-intercepts** are $(\mathbf{-2}, \mathbf{0})$, $(\mathbf{-1}, \mathbf{0})$, and $(\mathbf{3}, \mathbf{0})$

vi - For the range, find the **MAXIMUM**, which is also in the CALC menu.



The **range is**: $\{y \leq \mathbf{5.12}, y \in \mathbb{R}\}$

Note that the maximum is provided as an approximate value, to the nearest hundredth.

Class Example 2.12 *Characteristics of Polynomial Functions*

For each of the following polynomial functions, without using your graphing calculator, state:

i - The start and end behavior of the graph   ii - The number of possible x-intercepts

iii - Whether or not the graph will have a minimum or maximum point

iv - The domain of the function and the y-intercept

Use your graphing calculator to determine:

v - The x-intercepts of the graph   vi - The range of the function

(a) $\boldsymbol{y = x^3 - 2x^2 - 5x + 6}$

i - Start / end

ii - # of x-ints

iii - Max or min?

iv - Domain:

y-intercept:

v - Coords of x-ints:

vi - Range:

(b) $\boldsymbol{y = x^5 + x^4 - 7x^3 - 13x^2 - 6x}$

i - Start / end

ii - # of x-ints

iii - Max or min?

iv - Domain:

y-intercept:

v - Coords of x-ints:

vi - Range:

Class Example 2.13 *More Characteristics of Polynomial Functions*

For the function $\boldsymbol{p(x) = -x^4 - 3x^3 + 7x^2 + 15x - 18}$, without using your graphing calculator, state:

i - The start and end behavior of the graph

ii - The number of possible x-intercepts

iii - Whether or not the graph will have a minimum or maximum point

iv - The domain of the function and the y-intercept

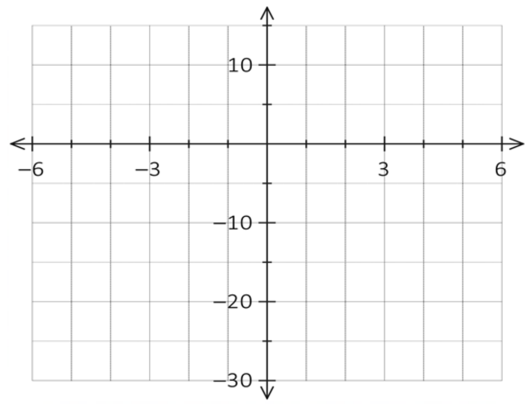
Then, use your graphing calculator to determine:

v - The x-intercepts of the graph

vi - The range of the function *nearest hundredth*

Lastly, **sketch** the graph on the grid below.

Label any intercepts and max / min points



Class Example 2.14 *Identifying Polynomial Functions*

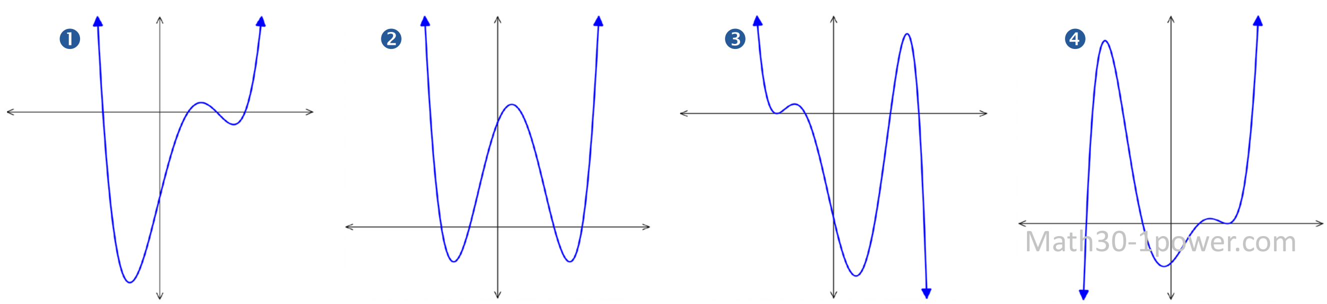
For each of the polynomial functions listed below, indicate the graph number that matches.

(a) $y = x^4 - 2x^3 - 7x^2 + 8x + 12$ ________

(b) $y = -x^5 + 11x^3 + 6x^2 - 28x - 24$ ________

(c) $y = x^5 - x^4 - 9x^3 + 13x^2 + 8x - 12$ ________

(d) $y = x^4 - 4x^3 - x^2 + 16x - 12$ ________



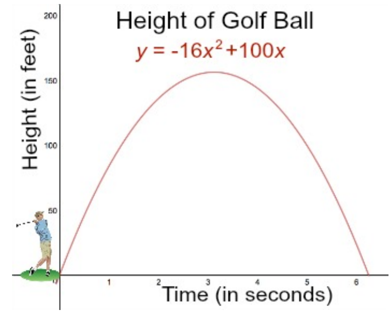
Answers from previous page

2.12 (a) **i** – Odd degree, Positive Leading Coefficient – So starts negative in quadrant III and ends positive in quad I.
ii – Degree 3 (odd), so between 1 and 3 x-intercepts. **iii** – Odd degree so no max / min
iv – Domain is $\{x \in \mathbb{R}\}$ y-intercept is: $y = (0)^3 - 2(0)^2 - -6(0) + 6$ ➔ $= 6$ So cords: $(\mathbf{0}, \mathbf{6})$
v – x-intercepts (use ZERO on calc): $(-\mathbf{2}, \mathbf{0})$, $(\mathbf{1}, \mathbf{0})$, and $(\mathbf{3}, \mathbf{0})$. **vi** – Range: $\{\boldsymbol{y} \in \mathbb{R}\}$.

(b) **i** – Odd degree, Positive Leading Coefficient – So starts negative in quadrant III and ends positive in quad I.
ii – Degree 5 (odd), so between 1 and 5 x-intercepts. **iii** – Odd degree so no max / min
iv – Domain is $\{x \in \mathbb{R}\}$ y-intercept is: $y = (0)^5 + (0)^4 - 7(0)^3 - 13(0)^2 - 6(0)$ ➔ $= \mathbf{0}$ So cords: $(\mathbf{0}, \mathbf{0})$
v – x-intercepts: $(-\mathbf{2}, \mathbf{0})$, $(-\mathbf{1}, \mathbf{0})$, $(\mathbf{0}, \mathbf{0})$, and $(\mathbf{3}, \mathbf{0})$. **vi** – Range: $\{\boldsymbol{y} \in \mathbb{R}\}$.

Applications of Polynomial Functions

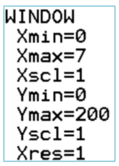
In Math 20, we saw how a certain type of polynomial function, the quadratic (degree 2) function, has applications in parabolic motion and finance. (To name just two!)



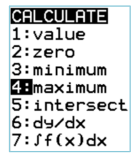
Above – classic math 20 question, do you remember how to find the **max height**?

Remember finding the ***maximum height of a ball***? →

First – match the window to what's given. Graph $y_1 = -16x^2 + 100x$

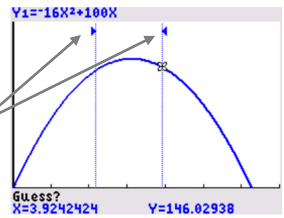


Then – from the CALC menu, select #4, "MAXIMUM"

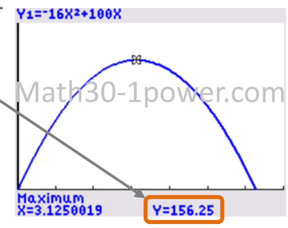


Next – For "left bound", hit enter anywhere to the left of the max. Do the same for "right bound", anywhere to the right.

The MAX should be between these arrows.



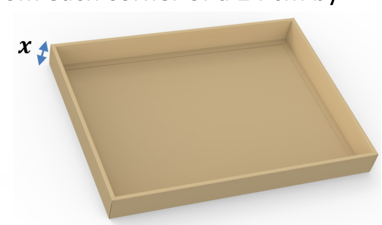
Finaly – Hit enter for "Guess". The max value is the y-**coord.**



So, the **maximum height** of the ball is **156.24 feet**, after **3.1 seconds.**

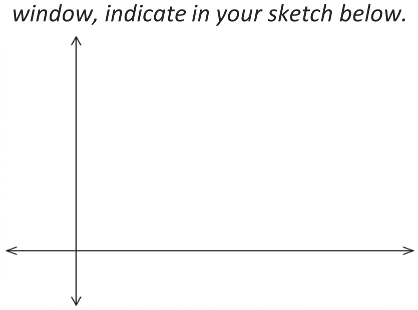
Class Example 2.15 *Constructing and Analyzing a Polynomial Equation*

A box is with no lid is made by cutting four squares (each with a side length "x" from each corner of a 24 cm by 12 cm rectangular piece of cardboard.



(a) Determine a function that models the volume of the box.

(b) Use technology to graph the function, and sketch below. Label each axis, provide a scale, and indicate any intercepts or max / min points. *Use your **graphing calculator** to obtain these… you'll need to "trial-and-error" a suitable viewing window, indicate in your sketch below.*



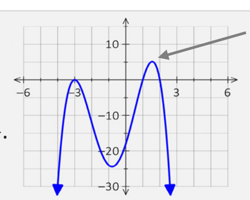
(c) State the domain of the function, with respect to the "real-world" constraints of the problem.

(d) State the value of "x" that gives the maximum volume. (Round to the nearest hundredth)

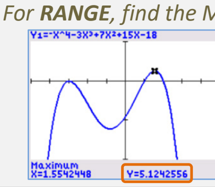
(e) State the maximum volume of the box, (Round to the nearest cm^3)

Answers from previous page

2.13 (a) **i** – Starts neg. in quad. III, ends neg. IV. **ii** – Can be 0 to 4 x-ints
iii – Even degree, neg lead coeff., so graph has MAX
iv – Domain is $\{x \in \mathbb{R}\}$ y-intercept is: $(0, -18)$
v – x-ints: $(-3, 0), (1, 0), (2, 0)$. **vi** – Range: $\{y \le 5.12, y \in \mathbb{R}\}$.



*For **RANGE**, find the MAX:*

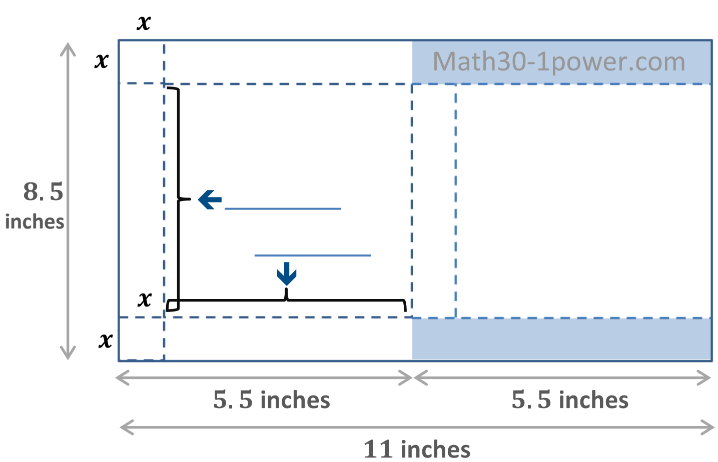


2.14 (a) ❷ (b) ❸ (c) ❹ (d) ❶ RTD Learning PowerMath

Class Example 2.16 *Constructing and Analyzing a Polynomial Equation*

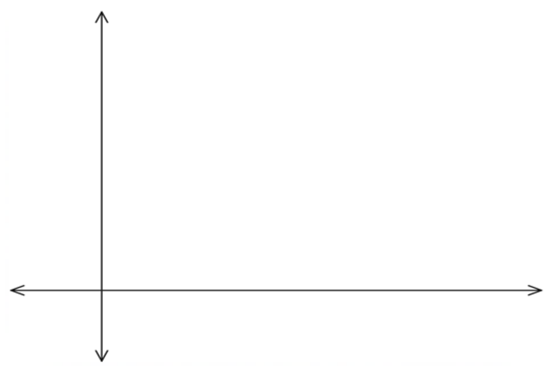
A box with a lid can be created by removing two congruent squares from one end of a rectangular 8.5 inch by 11 inch piece of cardboard. The congruent rectangles removed from the other end as shown. (The shaded rectangles represent the waste, or removed portions that will not be used in the box)

(a) In the diagram below there are two congruent rectangles; one that will form the **base** of the box, and one that will be the **top**. Complete the diagram by providing the missing dimensions (indicated with ➔/ 🡣) for the base and top.



(b) Determine a function that models the volume of the box.

(c) Use technology to graph the function, and sketch below. Label each axis, provide a scale, and indicate any intercepts or max / min points. *Use your **graphing calculator** to obtain these… you'll need to "trial-and-error" a suitable viewing window, indicate in your sketch below.*



(d) State the domain of the function, with respect to the "real-world" constraints of the problem.

(e) State the value of "x" that gives the maximum volume. (Round to the nearest thousandth)

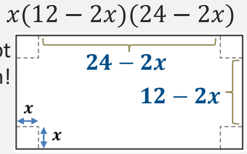
(f) State the maximum volume of the box, (Round to the nearest thousandth)

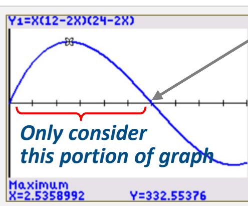
(g) State the dimensions that yield the maximum volume. (Round to the nearest thousandth)

Answers from previous page

2.15 (a) $V = x(12-2x)(24-2x)$

Note: Do not expand function!





Note that the part of the graph to the right of $x = 6$ is "irrelevant", as the y-coord (Volume) is negative.

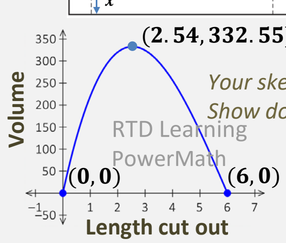
(c) **Domain is** $[0, 6]$

(d) **Max when** $x \approx 2.54$ **cm** *x-coord of MAX point*

(e) **Max Volume:** $\approx 333\ cm^3$ *y-coord of MAX*

Remember units!

(b)



Your sketch should only be shown within the domain, $[0,6]$.
*Show dots (●) on the graph at the domain start & end points, just **like here**.*

*Label the coords of the **intercepts** and **max point**.*
*Provide a scale, and label what **each axis represents**.*

2.1 Practice Questions

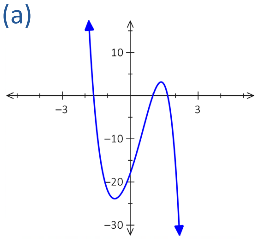
1. Indicate which of the following functions are polynomial functions:

(a) $y = 3x^5 - 3x^3 + 2x^2 + 11x + 6$ (b) $y = 3x^3 - 5x^{0.5} + 2$ (c) $y = 5$

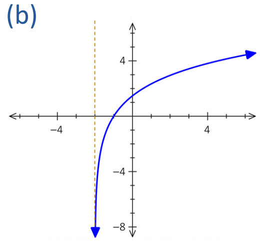
(d) $y = 4x^4 - 2x^2 + 5x^{-1} - 1$ (e) $y = 3x^3 - \sqrt{5}\,x$ (f) $y = x^2 + 5^x + 2$

2. Indicate which of the following graphs are likely those of polynomial functions:

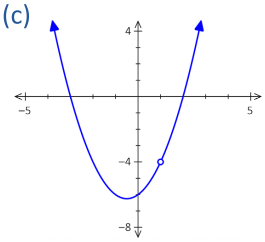
(a)



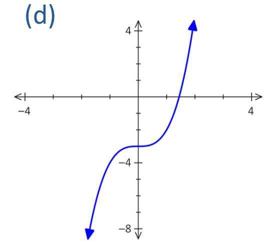
(b)



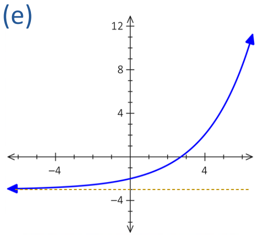
(c)



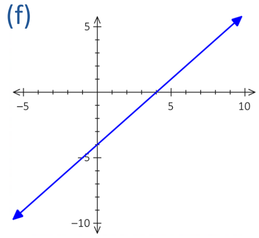
(d)



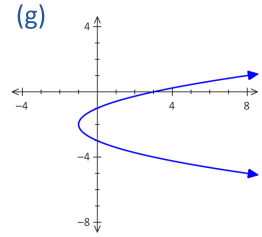
(e)



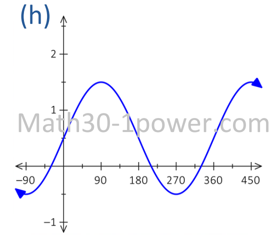
(f)



(g)



(h)



3. For each of the following polynomial functions, state each of the indicated characteristics. *Try as many as you can without graphing.*

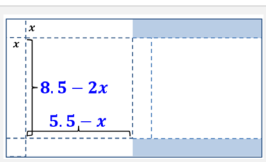
(a) $f(x) = x^3 + 8x^2 + 11x - 20$ (b) $y = 5 - x^4$

(c) $y = -2x^4 - 6x^3 + 14x^2 + 30x - 36$ (d) $y = -2(x+3)^2(x-2)^2(x-1)$

	(a)	(b)	(c)	(d)
i - Lead Coefficient	*i -* ______	*i -* ______	*i -* ______	*i -* ______
ii - Degree	*ii -* ______	*ii -* ______	*ii -* ______	*ii -* ______
iii - Start / end behavior	*iii -* ______	*iii -* ______	*iii -* ______	*iii -* ______
iv - Possible # of x-intercepts	*iv -* ______	*iv -* ______	*iv -* ______	*iv -* ______
v - Whether there is a max or min	*v -* ______	*v -* ______	*v -* ______	*v -* ______
vi - y-intercept	*vi -* ______	*vi -* ______	*vi -* ______	*vi -* ______

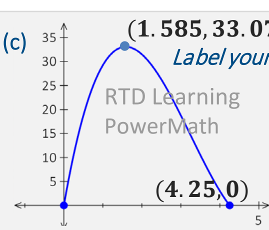
Answers from previous page

2.16 (a)



(b) $V = x(8.5 - 2x)(5.5 - x)$

(c)



Label your graph! And only show the relevant portion / within domain

(d) Domain is $[0, 4.25]$

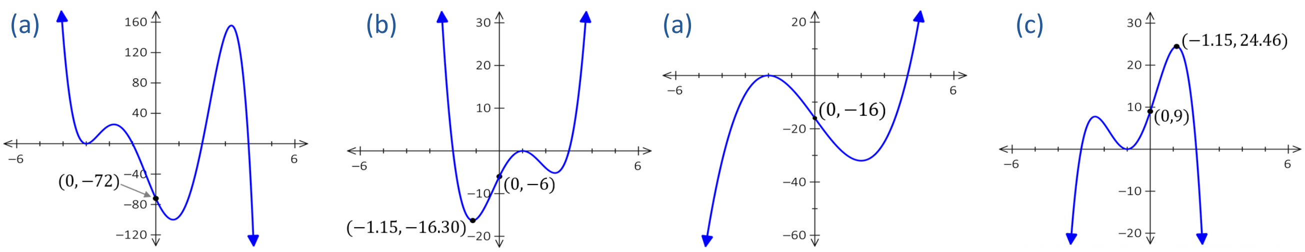
Remember units!

(e) Max when $x \approx 1.585$ in *x-coord of MAX point*

(f) Max Volume: $\approx 33.074\ in^3$ *y-coord of MAX*

(g) $3.915 \times 5.33 \times 1.585$ inches

4. For each of the following graphs, determine the indicated characteristics of the related function.



i - Is the degree even or odd? — (a) *i* - ______ (b) *i* - ______ (c) *i* - ______ (d) *i* - ______

ii - Is the leading coefficient pos (+) or neg (-) — *ii* - ______ *ii* - ______ *ii* - ______ *ii* - ______

iii - # of x-intercepts — *iii* - ______ *iii* - ______ *iii* - ______ *iii* - ______

iv - Range — *iv* - ______ *iv* - ______ *iv* - ______ *iv* - ______

v - Constant term in function equation — *v* - ______ *v* - ______ *v* - ______ *v* - ______

5. For each of the following functions, use technology to determine each of the indicated characteristics.

Note that using technology (graphing on your calc) is not required for each characteristic each time! For example, see if you can spot the x-intercepts of (c) without graphing. *(And degree and y-ints **can** always be found without graphing)*

Also note: To get best results graphing on your calculator – you must *practice setting your window!* For most of these you can use an **x-min of -6** and an **x-max of 6.** However for the **y min** and **max** …. use trial and error! (You'll want to see any relative max / min points, so ensure your window is "large enough")

(a) $f(x) = x^3 + 8x^2 + 11x - 20$ (b) $y = x^4 - 3x^3 - 12x^2 + 52x - 48$

(c) $y = -(x+3)^2(x-1)(x-3)$ (d) $y = -2x^2 + 2x + 24$

i - The degree — (a) *i* - ______ (b) *i* - ______ (c) *i* - ______ (d) *i* - ______

ii - The coordinates of any x-intercepts — *ii* - ______ *ii* - ______ *ii* - ______ *ii* - ______

iii - The coordinates of y-intercept — *iii* - ______ *iii* - ______ *iii* - ______ *iii* - ______

iv - The Range — *iv* - ______ *iv* - ______ *iv* - ______ *iv* - ______

Note: Where applicable, round to the nearest hundredth.

Answers to Practice Questions on the previous page

1. Polynomial functions are: (a), (c), (e) **2.** Polynomial functions are: (a), (d), and (f)

3.

	i	ii	iii	iv	v	vi
(a)	**1**	**3**	Starts neg in quad III, ends pos in quad I	**1** to **3**	No max or min	$(0, -20)$
(b)	**−1**	**4**	Starts neg in quad III, ends neg in quad IV	**0** to **4*** *see note 1*	Graph has a **max**	$(0, 5)$
(c)	**−2**	**4**	Starts neg in quad III, ends neg in quad IV	**0** to **4**	Graph has a **max**	$(0, -36)$
(d)	**−2**	**5*** *see note 2*	Starts pos in quad II, ends neg in quad IV	**3*** *see note 3*	No max or min	$(0, 72)$

Note 1: We can visualize this, as the graph of $y = x^4$ is similar to $y = x^2$, so visualize a "parabola" opening down and shifted 5 units up. So we know, without graphing, that there will be TWO x-intercepts!

$y = -2(x+3)^{2}(x-2)^{2}(x-1)$

Note 2: For functions in factored form, the degree of the entire function is the sum of all exponents, so: $2 + 2 + 1 = 5$

Note 3: Each factor corresponds to one x-intercept, so we know with certainty there are **3.** *There's an invisible "1" here!*

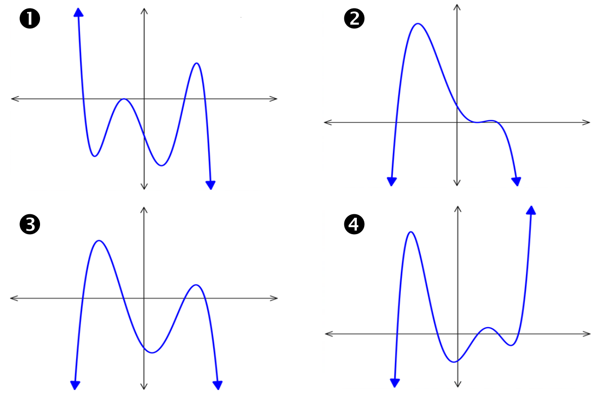
6. Without graphing (use your reasoning abilities!), match each of the following functions with its graph.

(a) $y = -x^5 + 12x^3 + 2x^2 - 27x - 18$ ______

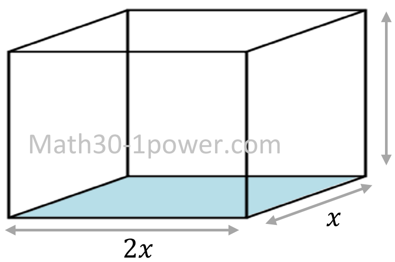
(b) $y = -x^4 + x^3 + 11x^2 - 9x - 18$ ______

(c) $y = x^5 - 2x^4 - 10x^3 + 20x^2 + 9x - 18$ ______

(d) $y = -x^4 + x^3 + 7x^2 - 13x + 6$ ______



7. A package may be sent through a particular mail service only if it conforms to specific dimensions. To qualify, the sum of its ***height*** plus ***the perimeter of its base*** must be no more than 72 inches. Also for our design, the base of the box (shaded in the diagram below) has a length equal to double the width.



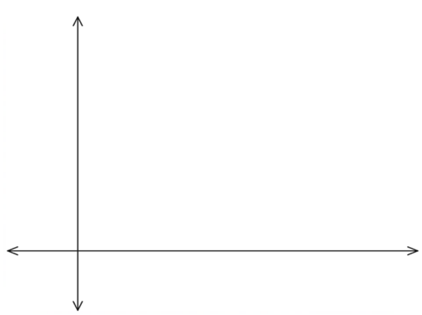
$h =$ ______

(a) In the blank on the left, state an expression for the height (h) of the box.
Need a hint? See the bottom of the next page.

(b) Determine a function that represents the Volume of the box.

(c) Use technology to graph the function obtained in (b) with a suitable viewing window.

Provide your sketch below, labeling any max/mins and intercepts. *Also fully label the axis, what each axis represents, and a suitable scale.*



(d) Provide a domain and range for your function obtained in (b), with respect to the "real world" constraints of the problem.

Domain: ______ Range: ______

(e) State the maximum volume of the box that can be sent.

(f) State the dimensions for the box that provides the maximum volume.

Answers to Practice Questions on the previous page

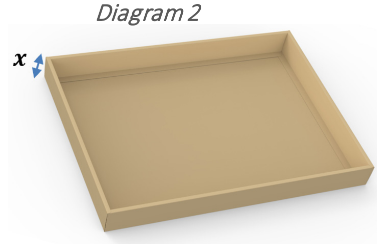
4.
(a) i **ODD** ii **NEGATIVE** iii **4** x-intercepts iv $\{y \in \mathbb{R}\}$ v Constant term: -72 *(represented by y-intercept)*
(b) i **EVEN** ii **POSITIVE** iii **3** x-intercepts iv $\{y \geq -16.30, y \in \mathbb{R}\}$ v Constant term: -6
(c) i **ODD** ii **POSITIVE** iii **2** x-intercepts iv $\{y \in \mathbb{R}\}$ v Constant term: -16
(d) i **EVEN** ii **NEGATIVE** iii **3** x-intercepts iv $\{y \leq 24.46, y \in \mathbb{R}\}$ v Constant term: 9

5.
(a) i 3 ii $(-5,0), (-4,0)$ and $(1,0)$ iii $(0,-20)$ iv $\{y \in \mathbb{R}\}$
(b) i 4 ii $(-4,0), (2,0)$ and $(3,0)$ iii $(0,-48)$ iv $\{y \geq -167.30, y \in \mathbb{R}\}$
(c) i 4 ii $(-3,0), (1,0)$ and $(3,0)$ iii $(0,-27)$ iv $\{y \leq 25.96, y \in \mathbb{R}\}$
Note: Each factor provides an x-intercept
(d) i 2 ii $(-3,0)$ and $(3,0)$ iii $(0,24)$ iv $\{y \leq 24.5, y \in \mathbb{R}\}$

8. An open box is to be made by cutting out squares from the corners of an 8 inch by 15 inch rectangular sheet of cardboard and folding up the sides.

Diagram 1

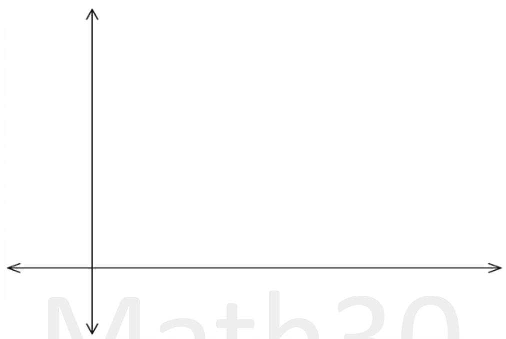
Diagram 2



(a) On diagram 1 on the right, provide expressions that represent the length and width of the finished box.

(b) Determine a function that models the volume of the box.

(c) Use technology to graph the function, and sketch below. Label each axis, provide a scale, and indicate any intercepts or max / min points. *Use your **graphing calculator**, provide a sketch below.*



(d) State the domain and range of the function, with respect to the "real-world" constraints.

(e) State the value of "x" that gives the maximum volume. (Round to the nearest hundredth)

(f) State the maximum volume of the box, (Round to the nearest in^3)

(g) Provide the dimensions that yield the box of maximum volume, (Round to the nearest hundredth)

Answers from previous and this page

HINT for #7(a): The perimeter of the base is: $2x + 2x + x + x = \mathbf{6x}$. As we wish for the largest volume box, we'll use all 72 inches (sum of perimeter and height) available. So $h + 6x = 72$, and $\mathbf{h = 72 - 6x}$.

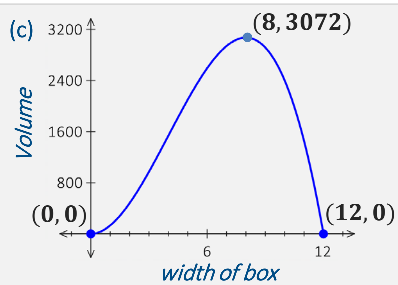
6. (a) ❶ (b) ❸ (c) ❹ (d) ❷

7. (a) $\mathbf{h = 72 - 6x}$

(b) $\mathbf{V = (2x)(x)(72 - 6x)}$

Graph $y_1 =$ in your calculator.
Trial-and-error to get best window.
Sketch should only show graph within domain. (between 0 and 6)

(c)



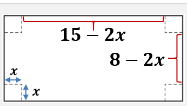
(d) **Domain is** $[0, 4.25]$
Range is $[0, 3072]$

Max when $x = 8$ inches

(f) **Max Volume:** $= 3072\ in^3$

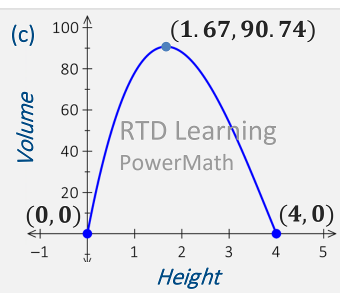
(g) $\mathbf{16}$ length $\times$ $\mathbf{8}$ width $\times$ $\mathbf{24}$ height **inches**

8. (a)



(b) $\mathbf{V = (15 - 2x)(8 - 2x)(x)}$

(c)



(d) **Domain is** $[0, 4]$
Range is $[0, 90.74]$

(e) **Max when $x = 1.67$ inches**

(f) **Max Volume:** $= 90.74\ in^3$

(g) $\mathbf{11.67}$ length $\times$ $\mathbf{4.67}$ width $\times$ $\mathbf{1.67}$ height **inches**